

# SCALE DEPENDENT DIMENSIONALITY

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## Abstract

We argue that dimensionality is not absolute, but that it depends on the scale of resolution, from the Planck to the macro scale.

## 1 Introduction

Is dimensionality dependent on the scale of resolution, or is it independent of this scale. This question becomes relevant in the light of some recent work (for example Cf.[1]). It has ofcourse been pointed out that the spin half character of a collection of Fermions leads to the usual three dimensionality of our space[2, 3], while the spin half itself is associated with the Compton wavelength as discussed in recent papers (Cf. for example[4]). Further, it was argued that as we approach the Compton scale, we encounter lower dimensionality[5, 6]. In this paper we point out that indeed the dimensionality is scale dependent.

## 2 Scale Dependence

We first notice that at Planck scale  $l_P$ , we have

$$N^{3/4}l_P \sim R \quad (1)$$

where  $N \sim 10^{80}$  is the number of elementary particles and  $R \sim 10^{28}cm$  the radius of the universe. This is not an empirical relation but rather can be

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deduced on the basis of a fluctuational creation of particles scheme recently discussed (Cf. for example[7, 8]). In this scheme,  $\sqrt{N}$  particles are fluctuationally created and this happens in the Compton time  $\tau$  of a typical elemental particle, a pion.

Further this corresponds to a fluctuational creation of  $N^{1/4}$  Planck particles as recently argued[9], in the Planck time  $\tau_P$ . Indeed, we have

$$\dot{N} \sim N^{1/4}/\tau_P \sim \sqrt{N}/\tau \quad (2)$$

Equation (2) leads to (1).

(1) shows that at the Planck length, the fluctuational dimensionality is 4/3. Interestingly this is the dimension of a Koch curve and a coastline [10]. With this dimensionality we should have

$$M \propto R^{4/3},$$

which indeed is true[11].

At the Compton scale of resolution, we have[7], as indeed can be deduced from (2) the well known Eddington formula,

$$R \sim \sqrt{N}l \quad (3)$$

(3) shows the two dimensional character at the Compton length. Indeed as noted in the introduction three dimensionality is at scales much greater than the Compton wavelength - as we approach the Compton wavelength we encounter two dimensionality as can be seen from (3) - indeed this was the key to explain puzzling characteristics of quarks including their fractional charge and handedness[6].

Finally at scales  $L \sim 10cm$ , we have

$$N^{1/3}L \sim R \quad (4)$$

(4) shows up the usual three dimensionality.

Interestingly, if we take the typical elementary particle the pion, and consider it successively as a 4/3 dimensional object at the Planck scale, a two dimensional object at the Compton scale and three dimensional at our macro scale, and consider successive densities

$$\rho_P \sim m/(l_P)^{4/3}, \rho_\pi \sim m/l^2 \quad \text{and} \quad \rho \sim m/L^3,$$

we have,

$$M \sim \rho_P R^{4/3} \sim \rho_\pi R^2 \sim \rho R^3,$$

as required.

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